sponding to the various values of λ. This series we call a "straight line." But we here make no implication of any geometrical character attached to the line. It is merely a collocation of points, where each point is a group of four symbols and nothing more.

Each point on the line is thus correlated with a specific value of the numerical magnitude λ, and thus if we have two points we may refer to them as the points λ and μ respectively.

As we are free from any geometrical meaning of our symbols, we can only introduce the expression "distance between two points" by defining precisely what is meant. The distance will be a function of λ and μ, whose form is to be decided by the properties which we desire to attribute to it. We may therefore select certain laws which we desire this function shall obey, and then discover what function will satisfy the conditions necessary.

We may take a hint from our familiar geometry as to the conditions to be imposed upon the distance function. If A, B, C be three points upon a straight line, then there is no more fundamental notion of distance than that implied in the equation

$$AB + BC = AC.$$

We shall accordingly insist that our distance function shall be obedient to this law (which we may call Law I.). If, therefore, λ, μ, ν be three points upon our straight line, and if $f(\lambda, \mu)$ denote the distance from λ to μ, then we find

$$f(\lambda, \mu) + f(\mu, \nu) = f(\lambda, \nu),$$

and as μ must disappear from this equation we have

$$f(\lambda, \mu) = \phi(\lambda) - \phi(\mu).$$

The first step in the construction of an appropriate distance function has thus been taken, but we have still a wide range of indeterminateness, for $\phi(\lambda)$ may of course be any conceivable function of λ. It will therefore be competent for us to select some additional law and to insist upon obedience to it also.

Again we revert to our familiar geometry for a suggestion. In that geometry it is assuredly obvious that the distance between two points cannot be zero unless the two points are coincident. Trite as this condition may appear, it is yet sufficient to clear every trace of indeterminateness from the form of ϕ: we shall term this Law II.

Combining Laws I. and II. it will be easy to show that if P be a point on the line then there can only be one point, Q, on the line at a given distance from P, for, suppose that there was a second point, Q', then we have, by Law I.,

$$PQ + QQ' = PQ';$$

but if PQ be equal to PQ', then

$$QQ' = 0,$$

from which, by Law II., we see that Q and Q' must be identical.

If the point P be defined by λ, and the point Q at a given distance therefrom be defined by μ, then the relation between λ and μ must be of the one-to-one type. The distance given, we must therefore have some equation of the form

$$A\lambda\mu + B\lambda + C\mu + D = 0.$$

Any constant values for A, B, C, D will be consistent with the conditions, but we can without loss of generality simplify this equation. If we make $\lambda = \mu$ we obtain the quadratic

$$A\lambda^2 + (B + C)\lambda + D = 0.$$

We thus see that there are in general two critical points on the line corresponding to the roots of this equation. If we choose these two points for x and y, which is of course possible without sacrifice of generality, the roots of this equation should be 0 and ∞, or in other words the constants A and D must be each zero. We thus see that by an appropriate choice of the fundamental points the relation between λ and μ assumes the simple type

$$B\lambda + C\mu = 0,$$

or, finally,

$$\lambda = h\mu,$$

in which h is a function of the particular distance between λ and μ.

We have, however, seen that the distance is also to be expressed in the form

$$\phi(\lambda) - \phi(\mu).$$

This must therefore be a function of h, that is, of $\lambda \div \mu$, and thus we have

$$\phi(\lambda) - \phi(\mu) = F\left(\frac{\lambda}{\mu}\right)$$

From this equation the particular value of the distance has disappeared. It must therefore be true for all values of λ and all values of μ. It must remain true if differentiated either with respect to λ or μ. We therefore have

$$\phi'(\lambda) = \frac{1}{\mu}F'\left(\frac{\lambda}{\mu}\right)$$

$$-\phi'\mu = -\frac{\lambda}{\mu^2}F'\left(\frac{\lambda}{\mu}\right),$$

whence

$$\lambda\phi'(\lambda) = \mu\phi'(\mu),$$

but as λ and μ are independent this requires

$$\phi'(\lambda) = \frac{H}{\lambda},$$

or

$$\phi(\lambda) = H \log \lambda,$$

whence finally we see that the distance between the two points λ and μ is

$$H \log \frac{\lambda}{\mu},$$

where H is a constant.

There seems nothing arbitrary in this process. We have set out with the two laws I. and II., and we have without any other assumption been conducted to the logarithmic conception of distance which lies at the foundation of the elliptic geometry. We might almost be tempted to ask how any other conception of distance can be reasonable. The two laws assumed are obviously true on any intelligible conception of distance, and yet they conduct to the logarithmic expression and apparently to nothing else.

It remains to show where the assumption made in ordinary geometry comes in. Hitherto we have not restricted the generality of the constants A, B, C, D which enter into the equation between λ and μ. Euclid, however, demands that the expression

$$(B + C)^2 - 4AC$$

shall be equal to zero. This has the effect of rendering the quadratic equation a perfect square. The logarithmic theory is accordingly evanescent, and we have to resort to the specialised conception of ordinary distance.

ROBERT S. BALL

NOTES

THE Council of the Royal Society at their last meeting awarded the Copley Medal to Auguste Kekulé, of Bonn (For. Mem. R.S.), for his researches in organic chemistry, and the Davy Medal to Jean Servais Stas, of Brussels (For. Mem. R.S.), for his researches on the atomic weights. At the same meeting, Prof. D. E. Hughes, F.R.S., and Prof. E. Ray Lankester, F.R.S., were nominated for the Royal Medals—the former eminent for his electric researches, and the latter for his services to embryology and animal morphology. Her Majesty has since signified her approval of these nominations.

THE anniversary meeting and annual dinner of the Royal Society will take place on Monday next—St. Andrew's Day.

THE session of the Society of Arts was commenced on Wednesday last by a most important address by the President, Sir F. Abel, on Appliances used in Mines. We shall give this on a subsequent occasion.

SIR JOSEPH HOOKER, we learn, retires, after a tenure of twenty years, from the Directorship of Kew Gardens on the 30th of the present month.

THE Savilian Professor of Geometry, J. J. Sylvester, M.A., Hon. D.C.L., proposes to deliver a public lecture on Saturday, December 12, at 4.45 p.m., in the Mathematical Lecture Room in the University Museum, Oxford, "On a General Theory of the Necessary Singularities of Curves of Unspecified Order." The lecture, although presupposing some elementary knowledge of modern algebra, will not go into details of calculation, but will have for its principal object to bring to light the existence of a new world of algebraical forms, co-ordinate in extent and parallel in character, genesis, and laws of association with those which occur in the theory of invariants.

WE hear with regret that there is a possibility of the *Zoological Record* being discontinued after the close of the next (the 21st) annual volume, unless additional support be received from those most intimately concerned in the welfare of this useful and (to zoologists) indispensable *résumé* of the work done in each year. It will be a standing disgrace to British, Colonial, and American zoologists, if they allow the *Record* to lapse on attaining its majority. Those who have worked with it cannot but acknowledge the aid it has afforded them; those who work without it run the risk of finding themselves anticipated. There are probably many local societies, public libraries, institutions, and private individuals that have not yet supported it, but should do so. The *Zoological Record* Association consists of members and subscribers. Members render themselves liable to the extent of 5*l.*, and on the exhaustion of this sum can withdraw or renew their membership. They receive the annual volume, and the average cost to them has at present amounted to about 24*s.* Subscribers pay annually 1*l.*, for which they receive the volume, and incur no further liability. All those who are interested in the continuance of our *Zoological Record*, and who are not already amongst its supporters, should lose no time in communicating with the Secretary of the *Zoological Record* Association, Mr. H. T. Stainton, F.R.S., Mountsfield, Lewisham, London, S.E.

THE Elizabeth Thompson Science Fund, which has been established by Mrs. Elizabeth Thompson, of Stamford, Connecticut, "for the advancement and prosecution of scientific research in its broadest sense," now amounts to $25,000. As the income is already available, the trustees desire to receive applications for appropriations in aid of scientific work. This endowment is not for the benefit of any one department of science, but it is the intention of the trustees to give the preference to those investigations, *not already otherwise provided for*, which have for their object the advancement of human knowledge, or the benefit of mankind in general, rather than to researches directed to the solution of questions of merely local importance. Applications for assistance from this fund should be accompanied by a full statement of the nature of the investigation, of the conditions under which it is to be prosecuted, and of the manner in which the appropriation asked for is to be expended. The applications should be forwarded to the Secretary of the Board of Trustees, Dr. C. S. Minot, 25, Mount Vernon Street, Boston, Mass., U.S.A. The first grant will probably be made early in January, 1886.

LA Société de Physique et d'Histoire naturelle de Genève offers a prize of 500 francs for the best unpublished monograph on a class or family of plants. The prize was founded by A. P. de Candolle. Manuscripts may be written in Latin, French, German (in Roman letters), English, or Italian, and should be sent in to the President of the Society before October 1, 1889, to the Athenæum, Geneva. Members of the Society are not permitted to compete, and the prize may be reduced, or not awarded at all, in the event of the essays being insufficient, or not conforming to these rules. The Society hope to be able to give the successful monograph a place in their *Transactions*, should that mode of publication be agreeable to the author.

TELEGRAMS from Madrid state that earthquake shocks have again occurred at Velez Malaga, on the coast of the province of Malaga. There appears also to be some seismic movement on the opposite African coast, producing landslips of considerable magnitude close to the Mediterranean, while the bed of the sea at the points affected is reported to have sensibly risen. There have also been sharp shocks of earthquake in Andalusia, especially at Alhama, which suffered so much last year.

DR. FOREL writes that the following shocks of earthquake have been observed in Western Switzerland:—November 15, 2h. 15m. a.m., at Sion, Gryon, Ollon, or the same region which was affected on September 26 last; November 18, 9h. 25m. p.m., Chevroux, south-east bank of Lake Neuchâtel; November 20, 5h. 45m. a.m., at Gondo, on the south face of the Simplon.

SEVERAL great earthquake waves from the Pacific were observed at San Francisco on November 19 between 1 and 8 o'clock p.m. at intervals of thirty-five minutes.

BRIGADE-SURGEON AITCHISON, the naturalist with the Afghan Boundary Commission, arrived in England on the 23rd inst. He brought his numerous collections with him as far as Batoum, and there shipped them on an English steamer for London. He has succeeded in obtaining a very fine specimen of a tiger from Turkestan, which, if it should reach England alive, will be unique of its kind, as no living specimen of this animal from those districts has as yet been brought to Europe.

CAPT. MANGIN, the inventor of the system of optical telegraphy as now practised in the French Army, has died suddenly from an attack of apoplexy at the early age of forty-five.

THE committee for erecting a statue in commemoration of the late Dr. Broca has opened a public competition for this monument.

THE French Minister of War has granted new credits to the Meudon aëronauts for the construction of a larger balloon. Workmen are enlarging the shed for building the apparatus.

M. MENIER, the well-known electrical engineer and contractor, has purchased a large property, rue de Chateaud'un (Paris), and is rebuilding it on a new system. He will sell electric light to all the lodgers in the house at a reasonable rate. It is the first time this speculation has ever been tried in Paris.

WE understand that the Science and Art Department have given their sanction to the exhibits from the Buckland Museum Collection being retained by the South Kensington Aquarium authorities until the close of the Indian and Colonial Exhibition next year. A better position for them could not be found, more especially as the casts of fish hung upon the walls are in *bon accord* with the living specimens in the tanks.

THE Norwegian Forest Association, started in 1881, is making very good progress. At present there are about 250 members. The Association's journal for the current year, embracing some 300 pages, contains a number of important papers by the most eminent Norwegian writers on the subject of forests and forest culture. A meeting for the discussion of important subjects takes place annually. The question of preserving the old and cultivating new forests appears to be coming more and more to the front in Scandinavia, where the Government has now established several schools with plantations for the cultivation of young plants. In Sweden the children of the rural Board-schools are often employed on fine days in planting out young trees. Thus during last summer some thousand acres have been planted in a single parish alone. Of late years private individuals too have done a great deal to retrieve the deforestation which has been going on by planting new trees. In one single parish in Norway, for instance, a proprietor has planted on waste land no less than a quarter of a million of spruce, fir, and larch trees, all obtained from the Government nursery. The price of the young plants is one farthing, and only 10 per cent. of the plants die. After thirty years each is valued at 7*d.* in the ground. These are of course valuations in a country where both labour and timber are cheap.

We learn that Dr. J. G. Garson has just been elected a Corresponding Member of the Anthropological Society of Paris.

As a memorial to the late Sir Titus Salt, and in recognition of his benefactions to Saltaire, the Governors of the Salt Schools have decided to build a new Science and Art School, costing about 6000*l.* The building will be completely finished by May 15, on which day will be opened an important exhibition on the lines of the late International Inventions Exhibition. For this purpose the present buildings and a field of six acres will be utilised. The arrangement and supervision of the lighting and other electrical work have been intrusted to Messrs. Woodhouse and Rawson, of London.

In the last number of the *Bulletin* of the American Geographical Society, Mr. Ernest Ingersoll publishes a paper on the manner in which the settlement of North America has affected its wild animals. He takes in succession the customary divisions of animal life—mammals, birds, reptiles, fishes, and the almost countless invertebrates, and shows how far these prevailed geographically in historical times, and how they have now either disappeared altogether, or been driven northwards into the Canadian forests, or, in the case of fish, away from the coasts. Mr. Ingersoll thus ranges over the whole animal kingdom, and in every department he has to record destruction—in many cases wanton and useless—and disappearance. It is a most instructive and interesting paper.

The Sanitary Institute of Great Britain has just completed the preparation of a volume which will be of great interest to the statistical world, containing selections from the reports and writings of the late Dr. W. Farr. The selection of the papers and reports and the editing of this work have been undertaken by Mr. Noel A. Humphreys, of the Registrar-General's Office. The volume consists of 550 pages and is divided into six parts: (1) population, (2) marriage, (3) births, (4) deaths, (5) life tables, (6) miscellaneous. It has long been the source of much regret among students of vital statistics, and others practically interested in that branch of sanitary science, that from the form and manner of the publication of Dr. Farr's valuable papers on statistics they have not been generally available, being scattered over a long series of Blue-Books and other Reports. The object of the Institute in publishing the selection is to give those interested in the subject a ready means of studying the valuable writings and tables of that eminent statistician.

The additions to the Zoological Society's Gardens during the past week include a Rhesus Monkey (*Macacus rhesus* ♀) from India, presented by Mrs. Berry; two Black-eared Marmosets (*Hapale penicillata*) from South-East Brazil, presented by Miss L. M. Graham; two Emus (*Dromæus novæ-hollandiæ*) from Australia, presented by Lord Northesk; an Emu (*Dromæus novæ-hollandiæ*) from Australia, presented by Mr. A. Garrett Smith; a Cuvier's Podargus (*Podargus cuvieri*) from Australia, presented by Mr. Cromwell Collins; a Tawny Owl (*Syrnium aluco*), British, presented by Mr. Phillips; an Anaconda (*Eunectes murinus*) from Demerara, presented by Mr. G. H. Hawtayne, C.M.Z.S.; a Robben Island Snake (*Coronella phocarum*), a Hoary Snake (*Coronella cana*) from South Africa, presented by the Rev. G. H. R. Fisk, C.M.Z.S.

OUR ASTRONOMICAL COLUMN

The French Photographs of the Transit of Venus.—The measurement of the 700 photographs obtained at the various French stations during the transit of Venus, 1882, is about to be commenced. An office has been organised for the purpose, the necessary credit has been granted, and a measuring instrument, belonging to the Meudon Observatory and lent by M. Janssen, has been supplied. This will be replaced in January next by a smaller one by the same makers, MM. Brunner, Frères. The measurements, it is expected, will be completed in fifteen months.

The Absorption-Spectrum of Oxygen.—M. Janssen, continuing at the Meudon Observatory his important and difficult researches on the spectra of the gaseous constituents of the terrestrial atmosphere, has recently given (*Comptes rendus*, vol. ci. No. 14) a brief notice of the results he has obtained. The spectrum of an intensely brilliant light is viewed through a tube 60 m. in length containing oxygen, the pressure of the gas being constantly increased up to a pressure of 27 atmospheres. With the increase of pressure, dark lines or groups of lines appear. The first to appear are those groups in the red, which M. Egoroff, who was the first to observe them, considered to be the A and B of the solar spectrum. With higher pressures, and a more brilliant source of light, lines are suspected between A and B and between B and C. Lastly, with the greatest pressures three dark bands appear; one near α, one near D, but more refrangible, and one in the blue. The solar spectrum does not show any similar bands, which, therefore, can scarcely be ascribed to oxygen in the state in which it exists in our atmosphere.

The Apparent Enlargement of Celestial Objects near the Horizon.—M. Paul Stroobant has recently devoted a considerable amount of care to examining the cause of this well-known phenomenon. His experiences lead him to reject the theories most commonly received, that the appearance is due either to comparison with terrestrial objects, or to the "flattened arch" shape ascribed to the celestial vault. Experiments made with pairs of electric sparks in a lofty hall, showed that if the two sparks overhead were 100 mm. apart, the pair on a level with the eye, and equally distant from the observer, needed only to be 81·5 mm. apart to seem separated to a similar extent. Comparisons of various pairs of stars gave a similar result, and the following formula was found to represent the apparent size, G, of a celestial object, at any given altitude, H, when the size on the horizon was taken as 100:—

$$G = 100 - 19 \sin H.$$

Beside this relation, depending evidently on some physiological effect connected with the position of the head, M. Stroobant found that an increase in the brightness of an object caused an apparent diminution in its size, and *vice versâ*. The great apparent size of the moon at rising was therefore, he considered, largely due to its comparative faintness when near the horizon.

Nova Andromedæ and its Relation to the Great Nebula.—There seems to be considerable difference of opinion as to whether the new star is to be regarded as having a real physical connection with the nebula, or as being connected with it in appearance only. M. Trouvelot (*Comptes rendus*, vol. ci. No. 17) ably pleads for the latter view. Comparing the present aspect of the nebula with the chart he made of it in 1874, he finds two new stars in the central district, one being the present *Nova*, the other a star of the 13th or 14th magnitude, which precedes it by about 20s. But he believes that the nebula itself has undergone no change during the appearance of the *Nova*, the impressions to the contrary being, he thinks, due to the superior light of the star having overpowered for a time the surrounding portions of the nebula; so that the arguments founded upon these supposed or apparent changes lose their force. The 1874 chart shows some 1283 little stars, which by their feebleness and crowding present the characteristic features of the Milky Way, which indeed appears to extend somewhat beyond the nebula; and these stars also appear to become less and less numerous the farther the observer travels from the Milky Way.